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SHOCK WAVE DIFFRACTION ON A THIN WEDGE MOVING WITH A SLIP RELATIVE TO THE WAVE FRONT WITH IRREGULAR SHOCK INTERACTION*

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The flow generated by the motion of a thin wedge through the front of a plane shock wave of arbitrary intensity is investigated. The shock wave front is at some side slip angle to the wedge edge and at an angle slightly different from the right angle to the wedge plane of symmetry. The wedge velocity relative to the gas upstream of the shock wave is supersonic. Interaction of the shock wave with the weak compression shock attached to the wedge is assumed irregular. The range of input parameters (angle of side slip, Mach numbers of the shock wave and wedge) are indicated for all possible flow patterns under these conditions. Solution of the plane problem of shock wave diffraction over a thin wedge obtained by the author in /1/ is extended the case of three-dimensional flow.

A similar generalization of solution of the problem of shock wave diffraction over a stationary thin wedge /2/ was first considered by Chester /3/. A solution of the same problem as considered here, but under conditions of regular shock interaction was obtained by Smyrl /4/.

The solution of this problem contains, as particular cases, solutions of plane problems of shock wave diffraction over a moving toward it thin wedge (at zero side slip angle) and over a thin wedge overtaking it when in the considered here general problem the side slip angle is π.

1. Statement of the problem. Let a thin wedge move at constant supersonic velocity in a quiescent perfect gas in which, independent of it, propagates at constant velocity a plane shock wave of arbitrary intensity. Their relative motion in space depends on the direction and magnitude of velocities $W = a_\infty M_\infty$ of the wedge and $U = a_\infty M$ of the shock wave front in the quiescent gas. The shock wave generates downstream of its front a stream of gas at constant velocity $a_1 M_1$ (a_∞ and a_1 are the speeds of sound in the gas upstream and downstream of the shock wave front, respectively). The particular case of encounter motion when the shock wave front is parallel to the wedge edge was investigated in /1,4/. Below, an arbitrary angle between the shock wave front and the wedge edge is admitted. When that angle is neither zero nor π, the wedge edge intersects the shock wave front at point O (Fig.1) which moves simultaneously along the edge and in the front plane. Hence the part of the edge on one side of point O moves in the gas whose motion is induced by the shock wave, while the

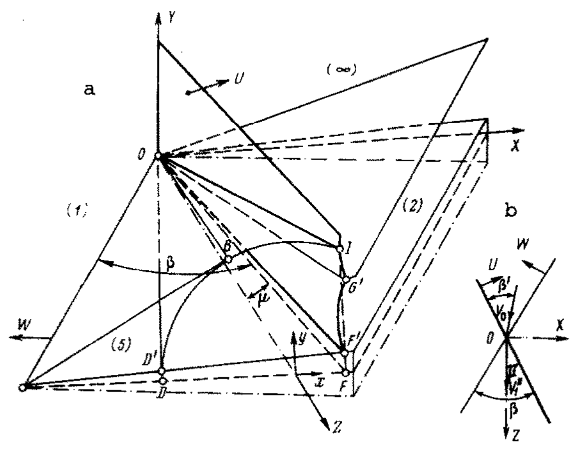

Fig.1

edge on the other side of that point moves through quiescent gas, so that the above statement

*Prikl.Matem.Mekhan.,44,No.2,258-270,1980 on the motion of the wedge and the shock wave will now relate only to that part of space in which the motion is not yet complicated by the penetration of the wedge edge through the plane of the shock wave front.

The fundamental parameters that define the flow are the Mach numbers M and M_∞, and the side slip angle β defined as the angle between the plane of the front of the unperturbed shock wave and the part of the wedge edge which moves through the gas whose motion is generated by the shock wave (Fig.1). As in /1/, we consider only flows in which the unperturbed shock wave front is at angle $\chi = \pi / 2 - \delta$ to the wedge plane of symmetry, which does not greatly differ from the right angle. The wedge motion relative to the gas downstream of the shock wave is at a low angle of attack δ and comprises side slip. The wedge half-aperture angle ε is of the same order of magnitude as δ.

In the reference system attached to point O the flow is stationary and the unperturbed shock wave may be considered as an oblique compression shock attached to some wedge whose side passes through the gas velocity vector downstream of that oblique shock. The considered here flow is assumed to differ only slightly from the motion of gas through such oblique compression shock. Particles of gas in such motion upstream of the shock wave translate in planes perpendicular to its front (Fig.1,b) at velocity $\mathbf{V}_0$ inclined to the shock wave front at the angle β' and equal in magnitude to V_0. After passing through the shock wave the gas particle velocity becomes $\mathbf{V}_1''$ and the angle between the vector of velocity V_1'' and the front becomes μ. Formulas defining the quantities V_0, β', V_1'', and μ appear in /4/.

The system of rectangular physical coordinates X, Y, Z has its origin at point O, the Z-axis is directed along the projection of the gas velocity vector on the plane perpendicular to the plane of the unperturbed shock wave front, and the Y-axis is perpendicular to it in the plane of the shock front (Fig.1,a).

The absence of a characteristic dimension in the problem enables us to assume the flow to be conical. Only that range of input parameters is considered for which the uniform unperturbed flow downstream of the shock wave considered in the selected reference system is supersonic, i.e. when $V_1'' > a_1$.

The region of nonuniform flow appears to be bounded by the Mach cone $OIBD'$, the wall, and the part $OF'G'I$ of the shock front. The cone apex is at point O and its axis is directed along vector $\mathbf{V}_1''$, i.e. along the Z-axis. The formula defining angle α between the cone generatrix and its axis is given in /4/. The plane which passes through the wedge edge and is tangent to the pertubation cone represents the front of a weak compression or rarefaction wave. Between it, the cone, and the wedge surface lies the uniform flow region (5) whose parameters do not greatly differ from those of region (1) of the unperturbed flow downstream of the shock wave lying above the surface $ONBI$.

The front of the weak compression shock attached to the wedge part situated ahead of the shock wave separates region (2) of the uniform flow parallel to the thin wedge surface from the region of the unperturbed uniform flow (∞) ahead of the shock wave. The following formulas:

$$a_2 = a_\infty\left(1 + \varepsilon\frac{\varkappa - 1}{2}\frac{M_\infty^2}{\sqrt{M_\infty^2 - 1}}\right), \quad p_2 = p_\infty\left(1 + \varepsilon\frac{\varkappa - 1}{2}\frac{M_\infty^2}{\sqrt{M_\infty^2 - 1}}\right), \qquad \mathbf{V}_2 = \mathbf{V}_0 + \varepsilon\frac{a_\infty M_\infty^2}{\sqrt{M_\infty^2 - 1}}\mathbf{k}$$

$$\mathbf{k} = \left\{-\frac{\cos(\beta - \mu)}{M_\infty};\ \frac{\sqrt{M_\infty^2 - 1}}{M_\infty};\ -\frac{\sin(\beta - \mu)}{M_\infty}\right\}, \qquad \mathbf{V}_0 = V_0\{-\sin(\beta' - \mu);\ 0;\ \cos(\beta' - \mu)\}$$

where $\varkappa$ is the adiabatic exponent, hold for the dimensional values of the speed of sound a_2, pressure p_2, and the velocity vector $\mathbf{V}_2$ in region (2) /4,5/.

The points N, B, I, F', and D' mentioned above lie in a plane normal to the Z-axis, and Fig.1,a corresponds to the case when point N of intersection of that plane with the edge lies downstream of the shock wave, i.e. when angle $\beta - \mu$ between the velocity vector $\mathbf{V}_1''$ and the part of the edge downstream of the shock wave is smaller than $\pi / 2$ and, furthermore, when the wedge velocity relative to the gas downstream of the shock wave $V_w = a_1 M_1 \cos\beta + a_\infty M_\infty$ is supersonic. Other patterns of flow are also possible, as shown in Fig.2 and 3. The dependence of the respective ranges of Mach numbers M and M_∞ on angle β is considered in Sect.3.

When the shock wave is of fairly high intensity owing to the increase of the speed of sound downstream of it at side slip angles $\beta \in [\beta_1, \pi / 2]$ (β_1 depends on $\varkappa$), the wedge velocity V_w relative to that gas may become subsonic. Then region (5) vanishes, the leading edge part ON lying downstream of the shock wave finds itself inside the Mach cone (Fig.2), and the flows under and above the wedge cease to be independent.

When angle $\beta > \pi / 2$, then at fairly high Mach numbers M_∞ of the wedge we may find that the angle $\beta - \mu > \pi / 2$. In that case point N of intersection of the plane normal to vector $\mathbf{V}_1''$ with the wedge edge lies upstream of the shock wave. The respective flow pattern is shown in Fig.3 for the case when the weak shock induced by the supersonic motion of the wedge in the gas downstream of the shock wave, separating region (5) from (1) is tangent to the shown part of the Mach cone surface $OD'B'I$, and not of its continuation beyond the line OI. In the opposite case, as the result of its intersection with the shock wave, a reflected shock OLB_1 (the straight line OL is in the shock wave front plane at an angle to line OF which is greater than that between OI and OF) tangent to the Mach cone is generated (Fig.3). Between this shock, the shock wave, and the cone surface a further uniform flow region (6) is created. In all cases of $\beta - \mu > \pi / 2$ only such values of input parameters are considered for which velocity V_w is supersonic.

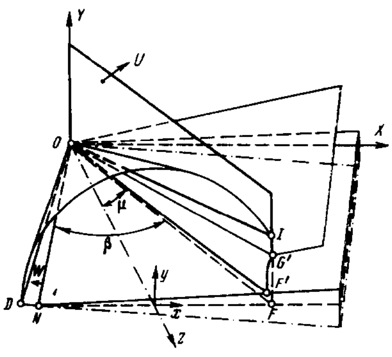

Fig.2

The diffraction of a shock wave over a wedge is complicated in all of the considered cases by its interaction with the weak shock induced by the supersonic motion of the wedge in the gas upstream of the shock wave. Whether the wave interaction is regular of irregular can be determined by projecting the velocity vector $\mathbf{V}_1''$ on line OG When the projection is larger or smaller than the speed of sound, the interaction is, respectively regular or irregular. In the first case a weak rarefaction shock tangent to the Mach cone is induced by the wave interaction. A solution of the respective boundary value problem for pressure, when the wedge velocity relative to the gas downstream of the shock wave is supersonic and the angle $\beta < \pi / 2$, was obtained in /4/.

Below, we consider the irregular interaction of waves, when the line OG' of intersection of shocks is in the perturbed part OIF' of the shock wave front. The surface of the weak tangential shock separates gas particles that have passed through part OIG' of the shock wave front from those that have first passed the weak compression shock front and, then, through the part $OG'F$ of the shock wave front. It was shown in /4/ that this surface does not affect the boundary value problem for pressure; it is not indicated in Figs. 1—3.

2. Self-similar coordinates. The limits of regularity. Analysis of the conical flow described above is carried out in a plane normal to the Z-axis. The position of the plane is chosen so that its intersection with the Mach cone is a circle of unit radius, with the corresponding dimensionless self-similar coordinates determined by formulas /3,4/

$$x = X / (Z \operatorname{tg} \alpha), \quad y = Y / (Z \operatorname{tg} \alpha) \tag{2.1}$$

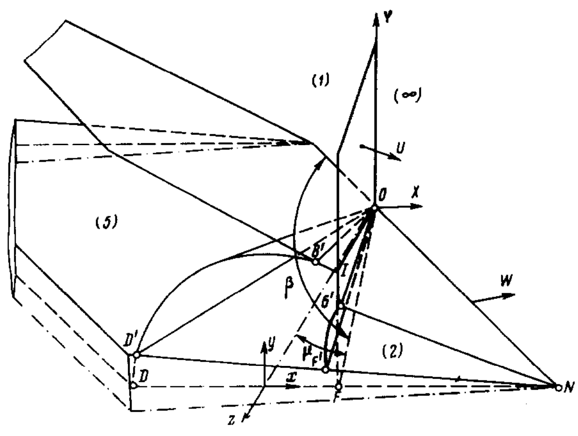

Fig.3

Using the formulas for V_1'', μ, and α it is possible to show that the introduced coordinates are transformed in self-similar coordinates as $\beta \to 0$, which were used in the analysis of the plane nonstationary problem /1,4/.

The cross sections of all three-dimensional flow patterns by planes normal to the Z-axis are shown in Fig.4. Owing to the smallness of ε and δ, section IF' of the perturbed shock front is replaced there by segment IF of the straight line parallel to the y-axis and section $D'F'$ by segment DF

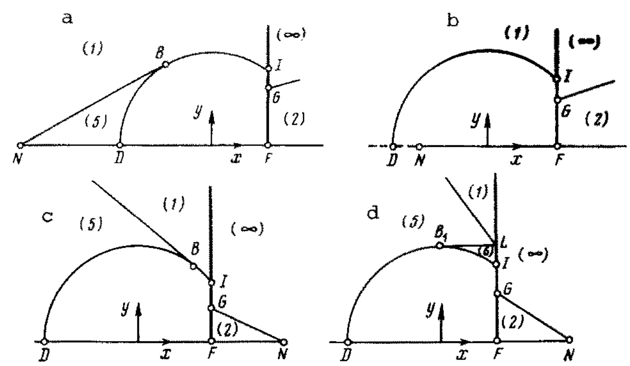

Fig.4

Dependence of coordinates of the characteristic points $I(x_0, y_0)$, $B(x_1, y_1)$, $F(x_0, 0)$, and $G(x_0, y_G)$ on the basic flow parameters is defined as follows:

$$x_0 = \frac{\operatorname{tg}\mu}{\operatorname{tg}\alpha} = m\frac{\cos\alpha}{\cos\mu}, \quad x_1 = -\frac{\cos(\beta-\mu)}{\cos\alpha\,(M_1\cos\beta + M_\infty a_\infty / a_1)}, \qquad y_i = \sqrt{1 - x_i^2} \quad (i = 0;\, 1) \tag{2.2}$$

$$y_G = \frac{\operatorname{tg}\mu + \operatorname{tg}(\beta-\mu)}{\operatorname{tg}\alpha}\frac{\cos(\beta-\mu)}{\sqrt{M_\infty^2 - 1}} = \frac{a_\infty}{a_1}\frac{M\cos\beta + M_\infty}{\sqrt{M_\infty^2-1}}\frac{\cos\alpha}{\cos^2\mu}$$

$$m = \{[2 + (\varkappa - 1)M^2] / [2\varkappa M^2 - (\varkappa - 1)]\}^{1/2}$$

These formulas also hold for the problem of regular wave interaction considered in /4/, but the formulas appearing there for y_G and x_1 are erroneous: the expression for y_G (denoted in /4/ by y_3) contains in the denominator the superflous multiplier $\cos^2(\beta - \mu)$, and the formula for x_1 is entirely wrong.

The expression for y_G can be resolved for M_∞, and obtain by the same token the equation of a set of curves in the coordinate plane M, M_∞, which, depending on parameter β, correspond to the specifed value of the coordinate y_G. In the particular case when point G coincides with point I, i.e. when $y_G = y_0$, such curve (for a given angle β) separates the region of variation of parameters M and M_∞ in two. One of these corresponds to the considered here irregular interactions (with $y_G < y_0$), the other to regular interactions when the solution derived in /4/ for $(y_G > y_0)$ is valid. The equation of the set of such curves (regularity boundaries) is of the form

$$M_\infty = M_\infty^{(0)} = \frac{M + c\sqrt{M^2 + c^2 - 1}}{c^2 - 1}\cos\beta, \quad \beta \neq \pi/2, \qquad c = \frac{a_1}{a_\infty}\sqrt{1 - m^2}$$

$$M = \{1 + [(\varkappa + 1)/(\varkappa - 1)]^{1/2}\}^{1/2}, \quad \beta = \pi/2$$

These lines may be, also, conveniently considered in coordinates λ, λ_∞ which represent ratios of velocities U and W to the critical speed of sound in the unperturbed gas upstream of the shock wave. The regularity boundaries are shown in Fig.5 by medium heavy solid lines. Curves 1–8 correspond to angles $\beta = 0, 40, 60, 80, 90, 100, 120, 180°$. In the λ, λ_∞ coordinates they all issue from one point (Fig.5,b), while in the M, M_∞ coordinates they have a vertical asymptote $M = \{1 + [(\varkappa + 1)/(\varkappa - 1)]^{1/2}\}^{1/2}$ (Fig.5,a). The regions that correspond to irregular interactions are to the right of these curves.

3. The range of input parameters for possible types of flow. Various flow modes may be generated depending on the values of input parameters M, M_∞, and β (Figs. 1—3). To each of these corresponds in the coordinate plane M, M_∞ and λ, λ_∞ a certain region dependent on β as on a parameter. The obtained in Sect.2 regularity boundaries enable us to determine which of these modes are accompanied by regular and which by irregular shock interactions.

The described analysis is bounded by the condition that $V_1'' > a_1$ which ensures the existence of the Mach cone, and which can be expressed in the form of the inequality

$$M_\infty > M_\infty^{(1)} = c\sin\beta - M\cos\beta \tag{3.1}$$

It can be also shown that

a) the wedge velocity relative to the gas downstream of the shock wave is supersonic, i.e.

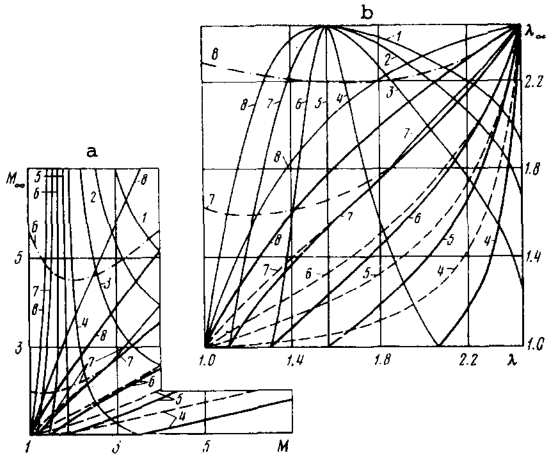

Fig.5

$V_w > a_1$, if

$$M_\infty > M_\infty^{(2)} = (1 - M_1 \cos\beta)\, a_1 / a_\infty \tag{3.2}$$

b) for $\beta > \pi/2$ the angle $\beta - \mu$ between vector $\mathbf{V}_1''$ and the wedge edge is smaller than $\pi/2$, when

$$M_\infty < M_\infty^{(3)} = [2(M^2-1)\sin^2\beta - (\varkappa+1)M^2] / [(\varkappa+1) M \cos\beta] \tag{3.3}$$

c) when $\beta - \mu > \pi/2$, i.e. $M_\infty > M_\infty^{(3)}$, condition $x_1 < x_0$ is satisfied if

$$M_\infty > M_\infty^{(4)} = -(2M^2-1)\cos\beta / M \tag{3.4}$$

The last condition means that the weak shock induced by the supersonic motion of the wedge in the gas in region (1) is tangent to the $OD'I$ part of the Mach cone surface (Fig.3). The comparison of (3.1) and (3.2) shows that $M_\infty^{(1)} \leqslant M_\infty^{(2)}$ and that equality is reached only when $M = M_*$, where

$$M_* = \{[2 - (\varkappa-1)\cos^2\beta] / [2\varkappa\cos^2\beta - (\varkappa-1)]\}^{1/2} \tag{3.5}$$

On the other hand, V_1'' and V_w can be simultaneously equal to the speed of sound a_1 only when vector $\mathbf{V}_1''$ is normal to the edge, i.e. when $\beta - \mu = \pi/2$. This means that $M_\infty^{(1)} = M_\infty^{(2)} = M_\infty^{(3)}$ when $M = M_*$; moreover, it follows from formula (3.4) that then $M_\infty^{(4)}$ is of the same magnitude, i.e.

$$M_\infty^{(1)} = M_\infty^{(2)} = M_\infty^{(3)} = M_\infty^{(4)} = (2M^2-1)\, m / M \quad \text{when} \quad M = M_*$$

Parameter M_* varies from 1 to ∞ as β increases in the range

$$\pi - \operatorname{arctg}[(\varkappa+1)/(\varkappa-1)]^{1/2} < \beta < \pi \tag{3.6}$$

In Fig.5 the dash line defined by the equation $M_\infty = M_\infty^{(2)}(M)$ corresponds to the condition of equality of V_w to the speed of sound, the dash-dot line $M_\infty = M_\infty^{(3)}(M)$ corresponds to the condition of perpendicularity of vector $\mathbf{V}_1''$ to the edge, and the thin continuous line issuing from the point $M_\infty = M_\infty^{(4)}(M)$ common to all curves corresponds to the condition $x_0 = x_1$ .The heavy solid line defines the lower boundary of possible values of M_∞ (or λ_∞) provided it is not equal unity. Curves *1—8* relate to angles $\beta = 0,\ 40,\ 60,\ 80,\ 90,\ 100,\ 120,\ 180°$.

If β is fixed so as to satisfy condition (3.6) (or $112.2° < \beta < 180°$ with $\varkappa = 1.4$), then the three curves $M_\infty^{(1)}(M), M_\infty^{(2)}(M), M_\infty^{(3)}(M)$ have a common point, with the first two tangent to each other (the second, the dash line curve always lies above the first) and are intersected by the third (the dash-dot) line that corresponds to condition $\beta - \mu = 90°$ (in Fig.5 this corresponds to angle $\beta = 120°$). Since for $\beta - \mu > 90°$ (i.e. above the dash-dot line) subsonic cases are not considered, hence the lower boundary of possible values of numbers M_∞ (the heavy line) for $M > M_*$ is defined by the equation $M_\infty = M_\infty^{(2)}$ and for $M < M_*$, by the equation $M_\infty = M_\infty^{(1)}$, since then $\beta - \mu < 90°$. The region between the heavy and the dash lines corresponds to the subsonic case (Fig.2) the region between the dash and the dash-dot lines to the supersonic case (Fig.1,a), that between the dash-dot and the continuous thin lines to the case of $\beta - \mu > 90°$ (Fig.3), and that between the thin continuous and the heavy lines corresponds to the reflection of a weak shock from the back of the shock wave (Fig.4,d).

As β increases the common point of these curves approaches the point at coordinates $M_\infty = M = 1$ $(\lambda_\infty = \lambda = 1)$. Angle $\beta = 180°$ corresponds to the plane problem of shock wave diffraction on an overtaking thin wedge and the entire heavy line is defined by the equation $M_\infty = M_\infty^{(3)}$, which means that the wedge velocity relative to the gas downstream of the shock wave must be supersonic. The region between the heavy and thin lines corresponds to cases of refraction of a weak shock in region (1) from the shock wave downstream of it, while the region to the left of the thin line corresponds to a flow without such reflection.

If angle β is contained in the interval $(\pi - \operatorname{arctg}\sqrt{(\varkappa+1)/(\varkappa-1)};\ \pi/2)$, the inequalities $M_\infty^{(4)} < M_\infty^{(1)} < M_\infty^{(2)} < M_\infty^{(3)}$ are satisfied for all M. Hence the entire heavy line is defined by the equation $M_\infty = M_\infty^{(1)}$ and the thin line is absent, i.e. there is no reflection of a weak shock from the shock wave downstream of it for these values of angle β (in Fig.5 the angle $\beta = 100°$ is within that interval).

If $\operatorname{arctg}\sqrt{(\varkappa+1)/(\varkappa-1)} < \beta < \pi/2$ (or $67.8° < \beta < 90°$ with $\varkappa = 1.4$), the dash-dot line is absent, i.e. always $\beta - \mu < 90°$.

It should be noted that in all so far described cases the medium heavy solid line (the regularity boundary) and the heavy line issue from the same point that corresponds to $M_\infty = 1$, and the first of these is always to the left of the second.

If $\arccos\sqrt{\varkappa(\varkappa-1)/2} < \beta < \operatorname{arctg}\sqrt{(\varkappa+1)/(\varkappa-1)}$ (or $58.0° < \beta < 67.8°$ with $\varkappa = 1.4$), the condition $M_\infty > M_\infty^{(2)}$ is satisfied for any $M_\infty > 1$ and, depending on whether condition $M_\infty > M_\infty^{(3)}$ is satisfied, either the supersonic (Fig.1,a) or the subsonic (Fig.2) case obtains. In Fig.5 angle $\beta = 60°$ corresponds to this interval, but because in this case $M_\infty^{(2)}$ exceeds unity only at very high numbers M (higher than approximately 40), the dash line defined by the equation $M_\infty = M_\infty^{(3)}$ is not shown.

Finally, when $0 < \beta < \arccos\sqrt{\varkappa(\varkappa-1)/2}$ the supersonic case obtains for any number M.

If the number $\varkappa \geqslant 2$, then at fairly high numbers M the number $M_\infty^{(3)} > 1$ for any angle β, i.e. we have the subsonic case when M_∞ satisfies the condition $1 < M_\infty < M_\infty^{(3)}$.

4. The boundary value problem (the supersonic case). The dimensionless perturbations of pressure p, density ρ, and the velocity vector components u and v along the x- and y-axes are defined by formulas /4/

$$p = \frac{\bar p - p_1}{\varepsilon \rho_1 a_1^2}, \quad \rho = \frac{\bar\rho - \rho_1}{\varepsilon \rho_1}, \quad u = \frac{\bar u}{\varepsilon a_1 \cos\alpha}, \quad v = \frac{\bar v}{\varepsilon a_1 \cos\alpha}$$

in which $\bar p, \bar\rho, \bar u$, and $\bar v$ are dimensional quantities inside the perturbation cone, and p_1, ρ_1, and a_1 in region (1).

The system of equations of gasdyanics for p, ρ, u, and v in the considered here three-dimensional problem coincide, after passing to self-similar variables and linearization, with the system of equations for the respective plane nonstationary problem /2/. After the elimination of ρ, u, and v, this system reduces to an equation of the elliptic type for function p in the region $IBDFI$. Along arc sections IB and BD the pressure is constant: $p = 0$ on IB and $p = p_5$ on BD, and

$$p_5 = \frac{q^2}{\sqrt{q^2-1}}\left(1 + \frac{M_1\delta/\varepsilon}{q}\right), \quad q = M_1\cos\beta + M_\infty a_\infty / a_1 \tag{4.1}$$

Condition $\partial p/\partial y = 0$ /2/ must be satisfied along section DF of the boundary.

Linearization of the laws of conservation at the shock wave whose front equation $x = x_0 + \varepsilon f(y)$ yields the following formulas for functions u, v, and p along section FI of the boundary of region $IBDFI$:

$$u = g_u(f - yf') + h_u\vartheta(y_G - y), \quad v = g_v f' + h_v\vartheta(y_G - y), \quad p = g_p(f - yf') + h_p\vartheta(y_G - y) \tag{4.2}$$

$$g_u = \frac{2}{\varkappa+1}\frac{\cos^3\mu}{\cos^2\alpha}\frac{\sqrt{M^2 + M_\infty^2 + 2MM_\infty\cos\beta}}{M_\infty + M\cos\beta} \times \left[\cos(\beta' + \mu) + \frac{\cos(\beta' - \mu)}{M^2}\right]$$

$$g_v = -\frac{M_1\cos\mu}{\cos\alpha}, \quad g_p = \frac{4M}{\varkappa+1}\frac{p_\infty}{p_1}\frac{a_1}{a_\infty}\frac{\cos^3\mu}{\cos\alpha}$$

$$h_u = \frac{2}{\varkappa+1}\frac{\cos\mu}{\cos\alpha}\frac{a_\infty}{a_1}\left\{\frac{M_\infty}{M^2}\cos\beta + M\left[\cos\beta - \frac{\varkappa+1}{2}\frac{\cos(\beta-\mu)}{\cos\mu}\right] - \frac{(\varkappa-1)M_\infty^2}{M}\right\}\frac{1}{\sqrt{M_\infty^2 - 1}}$$

$$h_v = \frac{M_\infty a_\infty}{a_1\cos\alpha}, \quad h_p = \frac{4}{\varkappa+1}\frac{p_\infty}{p_1}\left(\frac{M^2M_\infty^2}{2} - \frac{\varkappa-1}{4}M_\infty^2 + MM_\infty\cos\beta\right)\frac{1}{\sqrt{M_\infty^2 - 1}}$$

These formulas, after the elimination of function f yield the following relationship between perturbations of velocity and pressure components:

$$u = Ap + (h_u - Ah_p)\vartheta(y_G - y), \quad \frac{\partial v}{\partial y} = \frac{B}{y}\frac{\partial p}{\partial y} - \left(h_v - \frac{h_pB}{y}\right)\delta(y - y_G) \tag{4.3}$$

$$A = \frac{M^2\cos(\beta'+\mu)+\cos(\beta'-\mu)}{2M^2\cos\alpha\cos\beta'}\sqrt{\frac{2\varkappa M^2-(\varkappa-1)}{2+(\varkappa-1)M^2}}, \qquad B=\frac{\varkappa+1}{2}\frac{M^2-1}{2+(\varkappa-1)M^2}\frac{1}{\cos^2\mu}$$

where $(\delta(y-y_G)$ is the delta function.

As in the case of the plane problem worked-out in /1/, formulas (4.3) yield the boundary condition for function p when $x=x_0$; they differ only by the sign of the delta function in formula (2.2) in /6/ in which x_0 is to be substituted for m.

The second of formulas (4.3) yields, after integration along the front, as in the plane problem, the following normalization condition:

$$\int_0^{y_0}\frac{B}{y}\frac{\partial p}{\partial y}dy = h_v - h_p\frac{B}{y_G} - v_w, \qquad v_w=(M_1\cos\beta+M_\infty a_\infty/a_1+M_1\delta/\varepsilon)\sec\alpha \tag{4.4}$$

where the integral is taken in the meaning of its principal value.

Since in the supersonic case the form of the triangular region $IBDFI$, as well as the formulated in it boundary value problem for function p are of a form analogous to those in the plane problem of diffraction, hence the solution must be of the same form as that obtained in /1/. Final formulas for pressure distribution $p(x)$ on the wall and $p(y)$ along the shock front are the same as formulas (4.1) and (4.2) in /1/, except that the quantity K_0 is altered to

$$K_0 = -\frac{B}{y_G}\left(h_p+\frac{S}{b^2+1}\right)-\left(M_1\frac{\delta}{\varepsilon}+M_1\cos\beta\right)\sec\alpha$$

These formulas are written on the assumption that $\gamma_1, \gamma_2 > \sqrt{2}$ which in the plane problem could only be violated when $\varkappa > 5/3$, while in the presence of a side slip angle this can occur for any $\varkappa$. In such case, for example of $\gamma_2 < \sqrt{2}$, the alterations indicated in Sect. 4 of /1/ must be carried out. This applies also when $\gamma_1 < \sqrt{2}$.

The singularities of pressure distribution along the shock front which were discovered in the plane problem, i.e. the finite discontinuity and the logarithmic singularity of function p at point G, apparently characteristic of the irregular interaction of a weak shock with a shock wave of finite intensity, are present in the considered here three-dimensional flow.

As already indicated in Sect.1 when the angle $\beta-\mu>\pi/2$, reflection from the shock wave of a weak shock induced by the motion of a wedge in the gas downstream of the shock wave, is possible; the reflected front is then tangent to the Mach cone. In this case the pattern of flow in the x, y-plane (Fig.4,d) contains a new region (6) bounded by the reflected shock, the shock wave, and arc IB_1; the coordinates of point $B_1(x_1, y_1)$ are determined by formulas

$$x_1' = \frac{x_0 - y_L\sqrt{y_L^2-y_0^2}}{x_0^2+y_L^2}, \quad y_L=\frac{1-x_1x_0}{\sqrt{1-x_1^2}}, \quad y_1'=\sqrt{1-x_1'^2}$$

The perturbation of pressure p_6 in that region can be determined using the previously obtained coefficient of reflection of a weak wave from a shock wave of arbitrary intensity (see /7/). It is necessary to substitute $p_5 - p_6$ for p_5 in coefficients c_3 and c_4 in formulas (4.1) and (4.2) of /1/, and to add to the right-hand side of the expression for pressure perturbation $p(y)$ along the shock front the quantity p_6 as an addend.

5. The shape of the shock front. The third of conditions (4.2) makes it possible to determine, after the substitution into it of function $p(y)$, as defined by formula (4.2) in /1/, function $f(y)$ as the solution of an ordinary differential equation with the condition that $f=0$ when $y=y_0$. That solution is represented by the right-hand side of formula(5.2) in /1/ multiplied by $\cos\alpha/\cos\mu$.

In the case of inner reflection of a weak shock from a shock wave, mentioned at the end of Sect.4, the latter is deflected along section LI (Fig.4,d) from its unperturbed position as the result of wave interaction; the boundary condition for function $f(y)$ at point y_0 also changes. Using the third of formulas (4.2) it can be written as

$$f(y_0) = -\frac{\varkappa+1}{4}\frac{\cos\alpha}{\cos^3\mu}\frac{a_\infty}{a_1}\frac{p_1}{p_\infty}\frac{p_6}{M}\left(1-\frac{y_0}{y_L}\right)$$

The quantity $f(y_0)$ must be added to the right-hand side of formula (5.2) in /1/ after its mulitiplication by $\cos\alpha/\cos\mu$.

Function $f(y)$ defines the shock wave shift from its unperturbed position in the x, y-plane in which the investigation is carried out. That shift is not the same as its deflection along the normal to the unperturbed front position, which is obtained from $f(y)$ by multiplying it by $\operatorname{ctg}\alpha/\cos\mu$.

6. The motion in which the edge is inside the Mach cone. In conformity with Sect.1 in the subsonic case only the NF part of boundary DF belongs to the wall (Fig.4,b), and the coordinate of point N on the x-axis is determined by the formula

$$x_N = -\frac{\mathrm{tg}(\beta-\mu)}{\mathrm{tg}\,\alpha} = -\left(M_\infty \frac{a_\infty}{a_1} + M_1\cos\beta\right)\frac{\cos\alpha}{\cos(\beta-\mu)}$$

It is convenient to separate, as in /3/, the unknown function p in two, viz., $p = p_s + p_a$, so that p_s is symmetric and p_a antisymmetric relative to the axis $y = 0$. The derivation of solution for function p_a is carried out as in /3/; the distribution of p_a along the wall is defined by formula (64) in /3/. Function p_s is zero along the Mach arc ID and along section DF satisfies the condition $\partial p_s / \partial y = x_N v_w^s \delta(x - x_N)$, $v_w^s = (M_1 \cos\beta + M_\infty a_\infty / a_1)\sec\alpha$

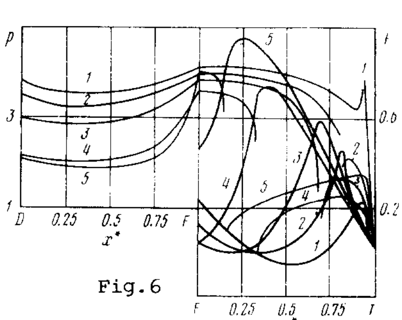

Fig.6

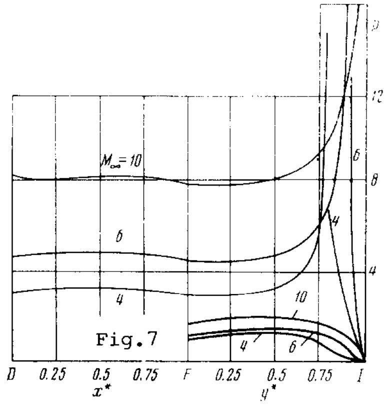

Fig.7

At the shock front function p_s satisfies the same condition as function p in Sect.4 and the normalization condition (4.4) where v_w^s is to be substituted for v_w. It can be shown that the distribution of p_s along the wall and the shock front is obtained from formulas (4.1) and (4.2) in /1/ by carrying out in terms with coefficients c_3, c_4, and d_3 the following substitutions: Arth for arctg, $\sqrt{2} - \gamma_3$ for $\gamma_3 - \sqrt{2}$, $\sqrt{2} - \gamma_4$ for $\gamma_4 - \sqrt{2}$, $2 - \gamma_3^2$ for $\gamma_3^2 - 2$, with $\gamma_3^2 = 2(x_0 - x_N)^2 / (1 - x_0 x_N)^2$. Instead of p_6 in coefficients c_3 and c_4 we must have now the quantity $(-x_N v_w^s / \sqrt{1 - x_N^2})$.

Thus in the subsonic case the pressure on the wall has at point N a logarithmic singularity, as well as a stronger singularity (when $\delta \neq 0$) of the type $1/\sqrt{\xi}$ as $\xi \to 0$. Pressure along the shock front has at point G the same singularities as in the supersonic case.

7. Results of calculations. The dependence of pressure on the wall on coordinate $x^* = (1 + x)/(1 + x_0)$, and of pressure along the front and the shape of the front on coordinate $y^* = y/y_0$ is shown in Figs.6 and 7, where the front shape is given by the heavy line, and all curves have been calculated for $\varkappa = 1.4$.

The effect of the side slip angle β on pressure distribution at the wall (the left-hand part of the diagram), and along the front, as well as on the front shape is shown in Fig.6 for the case of $M = M_\infty = 5$, $\delta = 0$. Curves *1–5* correspond, respectively, to angles $\beta = 0, 40, 60, 100, 120°$. For larger angles β the intersection line of fronts approaches the wall, inducing an increasing slope of curves along the part of the wall adjacent to the shock wave.

Figure 7 corresponds to the plane problem when angle $\beta = \pi$, which is the particular case of the flow taking place as the result of a thin wedge moving at supersonic velocity reaches the shock wave front from behind. In this case $M = 1.5$, $\delta = 0$. The respective Mach numbers M_∞ appear above each curve. When $M_\infty = 8$, the triple point G is very close to the point I $(y_G^* = 0.995)$, because of this the right-hand branch of the curve of pressure distribution along the front, which vanishes at point I and moves to infinity at point G, is not visible in the diagram. When $M_\infty = 10$ the wave interaction is regular and the curve representing pressure along the front, is continuous.

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